DAY —— 08 SEAT NUMBER

2024 VII 25 1100 **J-174** (E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. (8 Pages) Max. Marks : 80

General instructions :

*The question paper contains altogether 34 questions and is divided into **FOUR** sections.*

(1) ***Section A :*** *Q. **1** contains **Eight** multiple choice type of questions, each carrying **Two** marks.*

*Q. 2 contains **Four** very short answer type questions, each carrying **One** mark.*

(2) ***Section B :*** *Q. 3 to Q. **14** contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*

(3) ***Section C :*** *Q. **15** to Q. **26** contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*

(4) ***Section D :*** *Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*

(5) *Use of log table is allowed. Use of calculator is not allowed.*

(6) *Figures to the right indicate full marks.*

(7) *Use of graph paper is not necessary. Only rough sketch of graph is expected.*

(8) *For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabetical letter e.g. (a)........../ (b)........../ (c)............/ (d)............ etc.*

No mark (s) shall be given, if ONLY the correct answer or the alphabet of the correct answer is written.

Only the first attampt will be considered for evaluation.

(9) *Start answer to each section on a new page.*

SECTION – A

Q. 1. Select and write the correct answer for the following multiple choice type of questions : **[16]**

(i) $\cos\left[\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]=$ ____.

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\pi}{4}$ (2)

(ii) If θ is the angle between two vectors $\bar{a}$ and $\bar{b}$ and $|\bar{a}\cdot\bar{b}|=|\bar{a}\times\bar{b}|$ then θ is equal to ____.

(a) 0 (b) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π or $\frac{\pi}{6}$ (2)

(iii) The angle between the lines
$\bar{r}=(\hat{i}+2\hat{j}-3\hat{k})+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and
$\bar{r}=(5\hat{i}-2\hat{j}+7\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$ is ____.

(a) $\cos^{-1}\left(\frac{17}{21}\right)$ (b) $\cos^{-1}\left(\frac{20}{21}\right)$

(c) $\cos^{-1}\left(\frac{18}{21}\right)$ (d) $\cos^{-1}\left(\frac{19}{21}\right)$ (2)

(iv) The perpendicular distance of the plane
$\bar{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=5$ from the origin is ____.

(a) $\frac{5}{\sqrt{14}}$ units (b) $\frac{5}{14}$ units

(c) 5 units (d) $\frac{\sqrt{14}}{5}$ units (2)

(v) If $x = e^{\frac{x}{y}}$ then $\frac{dy}{dx} =$ ____.

(a) $1 - \frac{y}{x}$ (b) $1 + \frac{y}{x}$

(c) $\frac{x - y}{x \log x}$ (d) $\frac{x + y}{x \log x}$ (2)

(vi) $y = c^2 + \frac{c}{x}$ is solution of ____.

(a) $x^4 \left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} - y = 0$

(b) $x^2 \left(\frac{dy}{dx}\right)^2 + y = 0$

(c) $x^3 \left(\frac{d^2y}{dx^2}\right) - x\frac{dy}{dx} + y = 0$

(d) $x\frac{d^2y}{dx^2} = 4y$ (2)

(vii) Given that $X \sim B(n, p)$. If $n = 10$ and $p = 0.4$ then $E(X)$ and $Var(X)$ respectively are ____.

(a) 4, 0.24

(b) 0.4, 0.24

(c) 4, 2.4

(d) 3, 0.24 (2)

(viii) The approximate value of $\tan(44°30')$, given that $1° = 0.0175^c$, is ____

(a) 0.8952 (b) 0.9528

(c) 0.9285 (d) 0.9825 (2)

Q. 2. **Answer the following questions :** **[4]**

(i) Find the combined equation of the pair of lines $2x + y = 0$ and $3x - y = 0$ (1)

(ii) Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ (1)

(iii) Evaluate : $\int \frac{5^x}{3^x} dx$ (1)

(iv) Write the integrating factor (I.F.) of the differential equation $\frac{dy}{dx} + y = e^{-x}$. (1)

SECTION – B

Attempt any EIGHT of the following questions : **[16]**

Q. 3. If the statements p, q are true statements and r, s are false statements, then determine the truth value of the statement pattern :
$(q \wedge r) \vee (\sim p \wedge s)$ (2)

Q. 4. Find the inverse of matrix A by elementary row transformations, where $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ (2)

Q. 5. Find the polar co-ordinates of the point whose Cartesian co-ordinates are $(1, -\sqrt{3})$. (2)

Q. 6. Find the acute angle between the lines represented by $xy + y^2 = 0$. (2)

Q. 7. Using the truth table, show that the statement pattern $p \rightarrow (q \rightarrow p)$ is a tautology. (2)

Q. 8. If $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ then find the value of x, where $0 < 3x < 1$ (2)

Q. 9. Find the points on the curve given by $y = x^3 - 6x^2 + x + 3$ where the tangents are parallel to the line $y = x + 5$. (2)

Q. 10. Evaluate : $\int \frac{e^x(1+x)dx}{\sin^2(xe^x)}$ (2)

Q. 11. The displacement of a particle at a time t is given by $s = 2t^3 - 5t^2 + 4t - 3$. Find the time when acceleration is 14 ft/sec^2. (2)

Q. 12. Evaluate : $\int_0^{\pi/4} \sqrt{1 + \sin 2x}\, dx$ (2)

Q. 13. The probability distribution of X is as follows :

$X = x$	0	1	2	3	4
$P(X = x)$	0.1	k	$2k$	$2k$	k

Find (a) k

(b) $P(X < 2)$ (2)

Q. 14. Find the particular solution of :

$r\frac{dr}{d\theta} + \cos\theta = 5$ at $r = \sqrt{2}$ and $\theta = 0$ (2)

SECTION – C

Attempt any EIGHT of the following questions : **[24]**

Q. 15. In ΔABC, if $a \cos A = b \cos B$ then prove that the triangle is either a right angled or an isosceles triangle. (3)

Q. 16. Are the four points $A(1, -1, 1)$, $B(-1, 1, 1)$, $C(1, 1, 1)$ and $D(2, -3, 4)$ co-planar? Justify your answer. (3)

Q. 17. Find the difference between the slopes of the lines given by $(\tan^2\theta+\cos^2\theta)x^2-2xy\tan\theta+(\sin^2\theta)y^2=0$ (3)

Q. 18. Find the vector equation of the line passing through the point $(\hat{i}+2\hat{j}+3\hat{k})$ and perpendicular to the vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}+\hat{k}$. (3)

Q. 19. Let $\bar{a}$ and $\bar{b}$ be non-collinear vectors. If vector $\bar{r}$ is co-planar with $\bar{a}$ and $\bar{b}$ then prove that there exists unique scalars t_1 and t_2 such that $\bar{r}=t_1\bar{a}+t_2\bar{b}$. Hence find t_1 and t_2 for $\bar{r}=\hat{i}+\hat{j}$, $\bar{a}=2\hat{i}-\hat{j}$, $\bar{b}=\hat{i}-2\hat{j}$. (3)

Q. 20. Find the equation of the plane passing through the intersection of the planes $x+2y+3z+4=0$ and $4x+3y+2z+1=0$ and the origin. (3)

Q. 21. Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left(\sqrt{\frac{3-x}{3+x}}\right)$ (3)

Q. 22. Find the approximate value of $f(x)=x^3+5x^2-2x+3$ at $x=1.98$. (3)

Q. 23. Evaluate : $\int\frac{\sin(x+a)}{\cos(x-b)}dx$ (3)

Q. 24. Solve the differential equation $dr+(2r\cot\theta+\sin 2\theta)d\theta=0$ (3)

Q. 25. Let $X\sim B(10,0.2)$. Find

(a) $P(X=1)$

(b) $P(X\geq 1)$ (3)

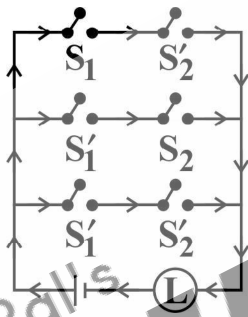
Q. 26. Find the expected value, variance and standard deviation of r.v. X whose p.m.f. is given as : (3)

$X=x$	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

SECTION – D

Attempt any FIVE of the following questions : **[20]**

Q. 27. Give an alternative arrangement for the following circuit, so that new circuit has minimum switches :



(4)

Q. 28. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ then find A^{-1} by Adjoint method. (4)

Q. 29. In ΔABC, D and E are points on BC and AC respectively such that $BD = 2DC$ and $AE = 3EC$. Let P be the point of intersection of AD and BE. Find ratio $\frac{BP}{PE}$ using the vector method. (4)

Q. 30. A firm manufactures two products A and B on which profit earned per unit are ₹3 and ₹4 respectively. Each product is processed on two machines M_1 and M_2. The product A requires one minute of processing time on M_1 and two minutes on M_2, while product B requires one minute on M_1 and one minute on M_2.

Machine M_1 is available for use not more than 450 minutes, while M_2 is available for 600 minutes during any working day.

Find the number of units of products A and B to be manufactured to get maximum profit. (4)

Q. 31. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is differentiable function of x such that the composite function $y = f[g(x)]$ is a differentiable function of x then prove that :

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Hence find $\frac{d}{dx}\left(\frac{1}{\sqrt{\sin x}}\right)$. (4)

Q. 32. Evaluate : $\int x^2 \sin 3x \, dx$. (4)

Q. 33. Prove that :

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)\,dx \text{ if } f \text{ is an even function}$$

$$= 0 \quad , \text{if } f \text{ is an odd function}$$

Hence find the value of $\int_{-1}^{1} \tan^{-1} x \, dx$. (4)

Q. 34. Find the area of the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Hence write area of $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (4)

◆◆◆